

Imbalanced magnetohydrodynamic turbulence modified by velocity shear in the solar wind

G. Gogoberidze • Y. M. Voitenko

Abstract We study incompressible imbalanced magnetohydrodynamic turbulence in the presence of background velocity shears. Using scaling arguments, we show that the turbulent cascade is significantly accelerated when the background velocity shear is stronger than the velocity shears in the subdominant Alfvén waves at the injection scale. The spectral transport is then controlled by the background shear rather than the turbulent shears and the Tchen spectrum with spectral index -1 is formed. This spectrum extends from the injection scale to the scale of the spectral break where the subdominant wave shear becomes equal to the background shear. The estimated spectral breaks and power spectra are in good agreement with those observed in the fast solar wind. The proposed mechanism can contribute to enhanced turbulent cascades and modified -1 spectra observed in the fast solar wind with strong velocity shears. This mechanism can also operate in many other astrophysical environments where turbulence develops on top of non-uniform plasma flows.

Keywords sun: solar wind - turbulence

1 Introduction

It is long known that the shear flows are important ingredient of the solar wind dynamics. Coleman (1968) was first who suggested that the solar wind is a turbulently evolving medium. He noticed that velocity-shear driven instabilities could produce power spectra

of magnetic and velocity fluctuations observed in the solar wind. He also suggested that the dissipation of the turbulence at high wave numbers could account for the anomalously high proton temperature observed in the solar wind at 1 AU.

The observations of the fast solar wind fluctuations (Bruno & Carbone 2013) show that below the ion-cyclotron frequency spectrum of the fluctuations consist of two intervals. Below the spacecraft-frame frequency $f_b \approx 10^{-3}$ Hz, which is usually referred as energy containing range, the spectral slope is close to -1 , while for higher frequencies the Kolmogorov spectrum is observed (this range is called inertial range). It is widely agreed that the formation of the Kolmogorov spectrum in the inertial range is related to the active turbulent cascade, as originally proposed by Coleman (1968), whereas the origin of the spectrum observed in the energy containing range is not entirely clear yet [see, e.g., Bruno & Carbone (2013) for a recent review]. First explanation was proposed by Matthaeus & Goldstein (1986). These authors suggest that the observed spectrum results from the superposition of uncorrelated samples of solar surface turbulence. Alternative possibility suggests that the formation of the spectrum can be related to the coronal dynamics (Matthaeus et al. 2007).

The viewpoint that velocity-shear driven instabilities could produce power spectra of magnetic and velocity fluctuations observed in the solar wind has two major shortcomings. Firstly, Belcher & Davis (1971) noticed that Alfvénic fluctuations in the fast flows of the solar wind are strongly imbalanced - the power of the Alfvén waves traveling outward from the sun is significantly larger than the power of inward propagating Alfvén waves and it is difficult to explain how the shear-driven instabilities can produce this asymmetry. Secondly, as mentioned by Bavassano et al. (1978), the Kelvin-Helmholtz instability cannot pro-

G. Gogoberidze

Institute of Theoretical Physics, Ilia State University, 3 ave. Cholokashvili, Tbilisi 0162, Georgia

Y. M. Voitenko

3 Solar-Terrestrial Centre of Excellence, Space Physics Division, Belgian Institute for Space Aeronomy, Ringlaan-3-Avenue Circulaire, B-1180 Brussels, Belgium

duce observed large-scale fluctuations. For these reasons it is widely accepted nowadays (Bruno & Carbone 2013) that the dominant, outward traveling Alfvén waves are mainly generated near the Sun below the Alfvénic critical point, as has been originally proposed by Belcher & Davis (1971).

On the other hand, even if generated, the inward waves can not propagate above the Alfvénic critical point and should have the local origin. Moreover, analysis of Helios and Voyager data (Roberts et al. 1987) showed that fluctuations in the solar wind become less imbalanced with increasing distance from the sun. Roberts et al. (1987) also found that the regions of strong shear are associated with a rapid evolution from the purely Alfvénic state to a more balanced state with accelerated turbulent cascade. These observations are puzzling in view of long known result of Parker (1964) that the Kelvin-Helmholtz instability is inefficient in the solar wind. The same result, strong enhancement of the turbulence cascade in strong shear flows, has been confirmed later in numerical simulations (Goldstein & Roberts 1995). Therefore, although enhancement of the turbulent cascade by the shear flows is seen both in the solar wind observations and numerical simulations, many aspects of its physics still remain unclear.

From the theory of neutral fluid turbulence it is long known (Tchen 1954; Hinze 1975) that the strong background shears can significantly affect turbulent dynamics. Namely, if the background velocity shear exceeds the velocity shears in turbulent fluctuations, then the distortion of fluctuations driven by the background shear dominates over nonlinear interactions. This leads to the enhancement of the turbulence cascade rate and formation of so-called Tchen spectrum $E(k) \sim k^{-1}$ (here k is a wave number and $E(k)$ is one dimensional spectrum of the fluctuations).

In this paper we consider strong incompressible imbalanced Alfvénic turbulence in the presence of background shear flows. By means of scaling analysis we show that, similarly to the fluid turbulence, the strong shear flow can significantly increase the energy cascade rate, resulting in the formation of the Tchen-like spectrum. This is especially crucial for the dominant Alfvén waves, because their evolution driven by the subdominant component is naturally weak because of the weak subdominant waves. Our analysis shows that this mechanism can explain strong enhancements of turbulent dynamics observed in the solar-wind shear flows.

The paper is organized as follows. Existing models of imbalanced magnetohydrodynamical (MHD) turbulence are reviewed in Sec. 2. Phenomenology of the Tchen model of strong imbalanced MHD turbulence in

the presence of strong velocity shear is developed in Sec. 3. Application of the obtained results to the solar wind turbulence is discussed in Sec. 4 and conclusions are given in Sec. 5.

2 Existing Models of Anisotropic Imbalanced MHD Turbulence

We consider incompressible MHD turbulence in the presence of the background magnetic field \mathbf{B}_0 . The Elsässer variables

$$\mathbf{w}^\pm = \mathbf{v} \pm \mathbf{b}/\sqrt{4\pi\rho}, \quad (1)$$

representing eigenfunctions of counter propagating Alfvén waves, are considered as the fundamental variables most useful to study MHD turbulence (Dobrowolny et al. 1980; Biskamp 2003). In equation (1) ρ is the mass density, \mathbf{v} and \mathbf{b} are velocity and magnetic field fluctuations respectively. The dynamics of the Elsässer variables is governed by the incompressible MHD equations

$$\left(\frac{\partial}{\partial t} \mp \mathbf{V}_A \cdot \nabla\right) \mathbf{w}^\pm + (\mathbf{w}^\mp \cdot \nabla) \mathbf{w}^\pm + \nabla p = 0. \quad (2)$$

Here p is the total (hydrodynamic plus magnetic) pressure and $\mathbf{V}_A \equiv \mathbf{B}_0/\sqrt{4\pi\rho}$ is the Alfvén velocity. In equations (2) we have neglected viscous and resistive dissipative terms, which become important on smaller scales.

Alfvén waves represent exact solutions of the ideal incompressible MHD equations. This means that if in equations (2), say, \mathbf{w}^- is zero initially, then $\mathbf{w}^+ = \mathbf{w}^+(x, y, z - V_A t)$ is a nonlinear solution of arbitrary form. Iroshnikov (1963) and Kraichnan (1965) realized that due to this property, the MHD turbulence can be described as nonlinear interactions of oppositely propagating Alfvén wave packets. The first model of MHD turbulence developed by Iroshnikov (1963) and Kraichnan (1965) assumed that the turbulence is isotropic. However, the mean magnetic field has a strong effect on the turbulence, in contrast to the mean flow in the hydrodynamic turbulence, which can be eliminated by the Galilean transformation. The anisotropy of MHD turbulence had been already seen in very early numerical simulations (Shebalin et al. 1983).

A theory of anisotropic balanced (under balanced we mean turbulence with equal energy of counter-propagating Alfvén waves) MHD turbulence was proposed by Goldreich & Sridhar (1995). This model implies that the dynamics of turbulence is dominated by the perpendicular cascade with respect to the mean

magnetic field whereas the parallel size of turbulent 'eddies' (wave packets) is determined by the critical balance condition. For wave packets with characteristic parallel length scales $\Lambda^\pm = \Lambda \sim 1/k_\parallel$ and perpendicular length scale $\lambda^\pm = \lambda \sim 1/k_\perp$, this condition implies that the characteristic time scale of wave packet collision Λ/V_A is equal to the characteristic time scale of the energy cascade $t_{cas} \sim \lambda/w_\lambda$, where w_λ is characteristic value of the Elsasser variables at scale λ . As a result one arrives at Kolmogorov-like phenomenology with $w_\lambda \sim \lambda^{1/3}$. Equivalently, for 1-dimensional perpendicular energy spectrum $E(k_\perp)$ we have $E(k_\perp) \sim k_\perp^{-5/3}$.

In the case of imbalanced MHD turbulence situation becomes more complicated. Assuming local turbulence, and noting that for Alfvén waves Elsasser fields w_λ^\pm are perpendicular to the mean magnetic field, it can be readily estimated that the nonlinear terms $(\mathbf{w}^\mp \cdot \nabla) \mathbf{w}^\pm$ are of the order $\sim w_\lambda^+ w_\lambda^- / \lambda$. Therefore, the straining rates for w_λ^\pm are (Lithwick et al. 2007; Chandran et al. 2009)

$$\omega_{sh}^\pm \sim \frac{w_\lambda^\mp}{\lambda}. \quad (3)$$

If typical parallel length scale of colliding wave packets is Λ , then characteristic timescale of their collision τ_{col} can be estimated as

$$\tau_{col} \sim \frac{\Lambda}{V_A}. \quad (4)$$

Note that if to packets of size Λ are counter-propagating with speed V_A , then collision time $\tau_{col} = \Lambda/(2V_A)$, but because we perform scaling analysis, this factor of 2 is ignored similar to other studies (Lithwick et al. 2007; Chandran et al. 2009).

We assume that w_λ^+ is the dominant component ($w_\lambda^+ \geq w_\lambda^-$). Dynamics of the turbulence depends on the dimensionless parameter

$$\chi^+ = \tau_{col} \omega_{sh}^+ \sim \frac{w_\lambda^+ \Lambda}{V_A \lambda}. \quad (5)$$

If $\chi^+ \gtrsim 1$, then subdominant wave packet is cascaded to smaller scale during one collision and we have the strong turbulence. Then for the energy cascade rate of the subdominant component we have

$$\varepsilon^- \sim (w_\lambda^-)^2 \omega_{sh} \sim \frac{(w_\lambda^-)^2 w_\lambda^+}{\lambda}. \quad (6)$$

This does not imply that the dominant wave packet is also cascaded during one collision.

Regarding the cascade of the dominant waves, various models give different predictions. Here we shortly consider main features and predictions of several recent

models of anisotropic imbalanced MHD turbulence. According to the model developed by Lithwick et al. (2007) the straining rate imposed by the subdominant waves on dominant ones, w_λ^-/λ , is imposed coherently over a time λ/w_λ^- and therefore cascade time for the dominant waves is

$$\tau^+ \sim \frac{\lambda}{w_\lambda^-}. \quad (7)$$

For the energy cascade rate of the dominant waves this equation gives

$$\varepsilon^+ \sim \frac{(w_\lambda^+)^2}{\tau^+} \sim \frac{(w_\lambda^+)^2 w_\lambda^-}{\lambda}. \quad (8)$$

According to the model developed by Chandran (2008), the strainings of the dominant waves by the subdominant ones are summed up randomly. This assumption makes cascade of the dominant waves weaker:

$$\varepsilon^+ \sim \frac{w_\lambda^+ (w_\lambda^-)^2}{\lambda}. \quad (9)$$

Yet another model of strong imbalanced MHD turbulence was developed by Beresnyak & Lazarian (2008). The key feature of this model is that the turbulent fluctuations of the dominant component cascade non-locally, from $k_{1\perp}$ to significantly larger $k_{2\perp}$ where $k_{2z(-)} = k_{1z(+)}$. As a result, the subdominant waves become more anisotropic than the dominant waves. This model predicts the cascade rate of dominant waves between the cascade rates predicted by two other models (equations 8 and 9), but there is no simple analytical expression for this cascade rate.

3 Tchen spectrum of MHD turbulence

Tchen (1954) was the first who recognized that the strong background shear can significantly affect the energy cascade rate and statistical properties of the hydrodynamic turbulence. In literature there exist several ways to obtain the Tchen spectrum, including spectral energy budget analysis (Tchen 1954), Heisenberg's eddy viscosity model (Katul et al. 2012) and scaling analysis (Perry et al. 1986).

Consider turbulent fluctuations of neutral fluid with characteristic excitation scale λ_f and amplitude u_f imposed in the mean flow with strong velocity shear, $S \equiv dV_0/dx \gg u_f/\lambda_f$. Then the distortion of a turbulent eddy by the background flow is stronger than the distortion by the turbulent flows (nonlinear interaction with other eddies). The main effect of the sheared mean flow is stretching the eddies along the flow, which in the

wave number space is equivalent to the increasing perpendicular (with respect to the mean flow) wave number. Consequently, the background shear flow transfers the energy to higher wave numbers faster than the nonlinear interactions.

If the fluctuations can be treated at outer scales as quasi-isotropic, then at some scale λ where the mean shear is greater than the inverse eddy turnover time v/λ , the effective cascade timescale shortens and becomes equal $\tau_{cas} \sim 1/S$ (although it has to be noted that nonlinear interactions are still necessary to ensure decorrelation of fluctuations and isotropic redistribution of fluctuation energy). If the energy cascade rate is denoted by ε , then from equation $\varepsilon \sim v_\lambda^2/\tau_{cas}$ we have

$$v_\lambda \sim \sqrt{\frac{\varepsilon}{S}}. \quad (10)$$

For one dimensional energy spectrum $E(k) \sim v_\lambda^2/k$ this gives

$$E(k) \sim \frac{\varepsilon}{Sk}. \quad (11)$$

Therefore, Tchen's model predicts that at relatively large scales, where the shear imposed by the turbulent fluctuations is still weaker than the mean flow shear, the energy spectrum should be inversely proportional to the wave number, $E(k) \sim k^{-1}$. When k increases, the shear associated with the turbulent eddies $s_\lambda \sim kv_\lambda$ also increases and starting from the wave number where $s_\lambda = S$ the turbulence is expected to follow Kolmogorov's phenomenology. There is significant evidence supporting Tchen spectrum both in boundary layer experiments and the atmospheric boundary layer measurements [see, e.g., Calaf et al. (2013) and references therein].

Here we develop an analogue of the Tchen phenomenology for the MHD turbulence. Consider incompressible imbalanced MHD turbulence is the presence of the background magnetic field $\mathbf{B}_0 \parallel \mathbf{z}$ and background shear flow $\mathbf{V}_0 = (0, 0, Sx)$. Linear dynamics of MHD waves in such a flow have been studied by Gogoberidze et al. (2004). Along with other phenomena (such as possibility of over-reflection and mutual transformation of different MHD modes), one of the main effects produced by the velocity shear is distortion of waves. In the wave number space it is equivalent to the linear variation in time of the perpendicular wave number, $k_x(t) = k_x - Sk_\parallel t$. Similarly to the hydrodynamic case, this is equivalent to the spectral transfer of energy in the perpendicular wave number space. Therefore, with strong velocity shear one can expect an enhancement of the cascade rate and formation of the Tchen-type spectrum in the MHD turbulence.

Here we consider the strongly imbalanced turbulence, the reason for which is twofold. First, the turbulence in the fast solar wind is strongly imbalanced, and there is plenty of in-situ observations to compare with our theoretical predictions. Second, in the imbalanced turbulence the cascade rate of the dominant component is reduced significantly because of the low amplitudes of subdominant waves responsible for the spectral transport in the dominant component. Consequently, even relatively weak background shear can strongly accelerate cascade in the dominant component.

Let us assume that the turbulence is excited isotropically at the (injection) outer scale λ_o with the characteristic amplitudes of dominant and subdominant components w_o^+ and w_o^- , respectively. Suppose that the background velocity shear is moderately strong, exceeding velocity shears in the subdominant component, but still smaller than the shears in the dominant component:

$$\frac{w_o^+}{\lambda_o} > S > \frac{w_o^-}{\lambda_o}. \quad (12)$$

In this case the cascade of subdominant waves is not significantly affected by the background shear and the spectral flux is still given by equation (6),

$$\varepsilon^- \sim \frac{(w_o^-)^2 w_o^+}{\lambda_o}. \quad (13)$$

On the contrary, the strainings of dominant waves by the background shear exceed the strainings imposed by the subdominant waves. Then, as in the Tchen fluid model, the cascade time for dominant waves is effectively shortened to $\tau_{cas}^+ \sim 1/S$ and the cascade rate is accelerated to $\gamma_{cas}^+ \sim 1/\tau_{cas}^+ \sim S$. In terms of this new cascade rate, the spectral flux in the wave number space at $k_\perp \sim 1/\lambda$ is given by

$$\varepsilon^+ \sim (w_k^+)^2 S. \quad (14)$$

Because of energy conservation, ε^+ is constant and all terms in this expression are k -independent, which results in the following one-dimensional wave number spectrum of energy:

$$E^+(k) \sim \frac{(w_k^+)^2}{k_\perp} \sim \frac{\varepsilon^+}{S} k_\perp^{-1}. \quad (15)$$

The relative strength of the cascades generated by the background and turbulent velocity shears can be conveniently described by the critical parameter

$$\eta_\lambda \equiv \frac{S\lambda}{w_\lambda}. \quad (16)$$

The cascade is dominated by the background shear and the -1 spectrum (15) is formed at scales where $\eta_\lambda > 1$.

The turbulent shears dominate at $\eta_\lambda < 1$ forming the $-5/3$ spectrum.

Equations (12-16) represent our model of the imbalanced MHD turbulence modified by the velocity shear. If the imbalanced MHD turbulence follows phenomenology by Lithwick et al. (2007), then formation of Tchen's spectrum is expected if the background shear is strong enough in sense of equation (12), i.e. when the cascade rate due to background shear ($\gamma_{cas}^+ \sim S$) is larger than the cascade rate due to the turbulent shears at the injection scale ($\gamma_o \sim w_o^-/\lambda_o$):

$$\eta_{\lambda_o} > 1. \quad (17)$$

In the cases where the turbulence follows phenomenology by Chandran (2008) with a weaker cascade of dominant waves, the Tchen spectrum can be formed by the proportionally smaller background shear (then the critical parameter η_λ should be modified correspondingly).

As the cascade generated by the background shear proceeds to smaller scales, the Tchen cascade rate S remain the same. On the contrary, the strainings imposed by the turbulent eddies become progressively stronger because of the stronger velocity gradients in the small-scale eddies. Then η_λ decreases below η_{λ_o} and the Tchen-type cascade eventually arrives to the spectral break

$$\lambda_b = \frac{w_b^-}{S}, \quad (18)$$

where $\eta_{\lambda_b} = 1$, i.e. the background and turbulent shears become the same. The Tchen wave number spectrum $\sim k_\perp^{-1}$ is formed at scales $\lambda_o > \lambda > \lambda_b$, whereas the strongly turbulent spectrum $\sim k_\perp^{-5/3}$ is formed at smaller scales $\lambda < \lambda_b$.

4 Application to the solar wind turbulence

Recent studies based on in-situ observations have revealed that the fast-slow solar wind interface has two parts: a smooth "boundary layer" surrounding the fast wind, and a sharper "discontinuity" between the slow and intermediate solar winds (Schwadron et al. 2005). A relatively strong velocity shear was observed by Ulysses over its first orbit in the transition area between the fast and slow solar winds at $13^\circ - 20^\circ$ latitudes. The data analysis (McComas et al. 1998) showed that the boundary layer separating two winds consists of two regions, the first one with the width $l_1 \approx 2 \times 10^7$ km and velocity difference $\Delta V_1 \approx 200$ km/s and the second one with $l_2 \approx 8 \times 10^7$ km and $\Delta V_2 \approx 100$ km/s.

The spectral brake between the "energy containing range" and the "inertial range" occurs at the spacecraft-frame frequency $f_b \approx \times 10^{-3}$ Hz (Telloni et al. 2015; Bruno & Carbone 2013). The corresponding break scale. It is well known that the power of inward propagating Alfvén waves in the fast solar wind streams is about one order of magnitude lower than the power of outward waves [see, e.g., Wicks et al. (2011) and references therein]. As the typical values in the fast solar wind we take $w_b^- \sim 7$ km/s for the subdominant wave amplitude at the scale λ_b (Wicks et al. 2011; Gogoberidze et al. 2012) and $V_{sw} \approx 600$ km/s for the solar wind speed (Bruno & Carbone 2013).

Noting that $\lambda_b = V_{sw}/f_b$ and $S = \Delta V_1/\Delta l_1$, with observed numerical values our model predicts

$$f_b = \frac{V_{sw}\Delta V_1}{\Delta l_1 w_b^-} \approx 1.2 \times 10^{-3} \text{ Hz}. \quad (19)$$

As we see performed rough estimate gives the value which is the same order of magnitude as the observed spectral brake frequency. Below f_b our model predicts the Tchen spectrum $\sim k_\perp^{-1}$. Although we do not claim that all observed $\sim k_\perp^{-1}$ spectra are generated by our mechanism, the correspondence between the model and observations is good enough to motivate further observational studies. In particular, as the break wave number $k_{\perp b}$ between $\sim k_\perp^{-1}$ and $\sim k_\perp^{-5/3}$ spectra is proportional to the background shear S , the presence of positive correlation between $k_{\perp b}$ and S in various data sets of fast solar wind streams would strongly support our mechanism.

As it is known (Bruno & Carbone 2013) the -1 spectrum is not observed in the slow solar wind. Therefore another interesting direction of further research is to study whether this phenomenon is related to the absence of strong velocity shear in the slow solar wind.

5 Conclusions

We developed a semi-phenomenological model of incompressible imbalanced MHD turbulence in the presence of sheared background flows. Our results can be summarized as follows:

1) The Tchen-type spectrum $\sim k_\perp^{-1}$ can be generated by the background velocity shear exceeding the shears of the subdominant Alfvén waves at the injection scale λ_o .

2) The k_\perp^{-1} spectrum breaks down at the scale λ_b given by (18), where the turbulent shears of the subdominant component become as strong as the background shear. The k_\perp^{-1} spectrum extends from λ_o to λ_b .

3) At smaller scales, $\lambda < \lambda_b$, the Kolmogorov $k_{\perp}^{-5/3}$ spectrum is formed by the turbulent velocity shears.

It is long known, but still unexplained, that in the fast solar wind streams the spectral index of turbulent fluctuations at large scales is close to -1 and the spectral break frequency is close to 10^{-3} Hz (see e.g. Marsch (1991)). These observations are compatible with the mechanism we propose here, which motivates its future verification by observations.

Our model can be applied to other astrophysical environments with strong velocity shears, like astrophysical jets and supernova explosions.

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